Main Question Addressed

Truthful mechanisms approximating the Liquid Welfare in Combinatorial Auctions with the same approximation guarantees as the state-of-the-art for the Social Welfare?

Combinatorial Auctions [No Budgets]

- Generate a lot of revenue (e.g., spectrum auctions)
- Important applications (e.g., airport slot auctions)

Social Welfare (SW) as the Efficiency Metric:

\[ SW = \sum_{i=1}^{n} v_i(S_i) \]

Truthful mechanisms for maximizing SW in CAs truthfully: extremely challenging!

- Instead, we resort to approximations for specific classes of valuation functions.
- Submodular Functions: \( S \subseteq T, x \notin T: v(S \cup \{x\}) - v(S) \geq v(T \cup \{x\}) - v(T) \)

State-of-the-art approximations of SW

Worst – Case
(no further assumptions apart from submodularity)

- \( O(\log m) \) [Krysta & Vocking, '12]
- \( O(\sqrt{\log m}) \) [Dobzinski, 2016]

Bayesian
(valuations drawn from known distributions)

- \( O(1) \) [Feldman, Gravin, Lucier, 2014; Duetting, Feldman, Kesselheim, Lucier, 2017]

- Posted price mechanisms used in both cases
- Bidders choose bundles through Demand Queries (DQ): 
  \( S_i := DQ(v_i, U_i, \vec{p}) = \arg\max_{T \subseteq U_i} \{v_i(T) - p(T)\} \)

Combinatorial Auctions [With Budgets]

Notion of welfare efficiency must balance between the bidder’s willingness (= valuation) and ability (= budget) to pay

Liquid Welfare (LW) [Dobzinski & Paes Leme, 2014]

\[ LW = \sum_{i=1}^{n} \hat{v}_i = \sum_{i=1}^{n} \min\{v_i(S_i), B_i\} \]

Our Approach

The Core Mechanism [e.g., Krysta & Vocking, ’12]

1. Fix bidder ordering \( \pi \) & set \( U_1 = U_2 = \cdots = U \)
2. Initial prices \( \vec{p}^{(1)} = (p_1^{(1)}, \ldots, p_m^{(1)}) \)
3. For each bidder \( i \sim \pi \) do:
4. Let \( i \) choose \( S_i := DQ(v_i, U_i, \vec{p}) \)
5. With prob. \( q \) give \( i: S_i & set: U_{i+1} = U_i \setminus S_i \)
6. Update prices \( \vec{p}^{(i+1)} = (p_1^{(i+1)}, \ldots, p_m^{(i+1)}) \)

Truthfulness (clear, due to choice of \( S_i \) from DQ)

- Appropriately choosing initial prices & price update rule:
  - \( O(\log m) \) - apx for SW in worst-case
  - \( O(1) \) - apx for SW in Bayesian settings

Lemma 1: If \( v: \text{submodular/XOS} \rightarrow \vec{v}: \text{submodular/XOS} \)

Why not change DQ in Step 4: \( DQ(\vec{v}, U_i, \vec{p})? \) Not truthful

Definition: Budget Compatible Demand Query (BCDQ):

\[ S_i := BCDQ(v_i, U_i, \vec{p}, B_i) = \arg\max_{T \subseteq U_i} \{v_i(T) - p(T)\mid p(T) \leq B_i\} \]

Truthfulness restored! What about approximations?

- Guarantees same order apx, just added constant of 1/2

Introducing Competitive Markets in CAs

\[ \Pr[\hat{OPT}_T \geq \left(1 - \frac{\epsilon}{2}\right) OPT] \geq 1 - \delta \]

- Homogeneity of the market
- Important buyers in both sets
- With high probability if you only sell in the second set, you don’t lose much