A Bridge between Liquid and Social Welfare in Combinatorial Auctions with Submodular Bidders

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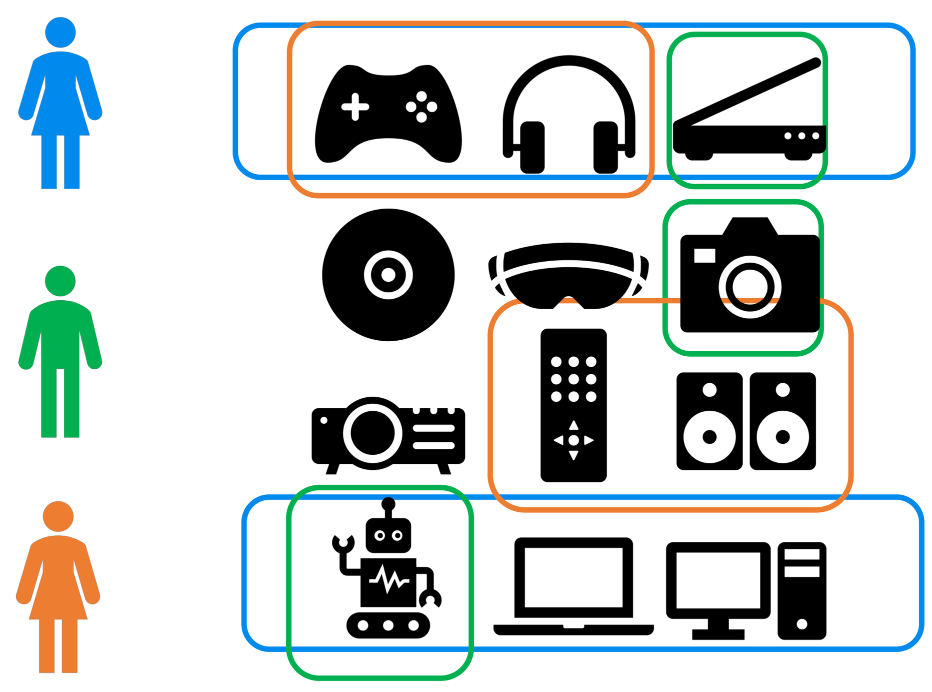
Main Question Addressed

Truthful mechanisms approximating the Liquid Welfare in Combinatorial Auctions with the same approximation guarantees as the state-of-the-art for the Social Welfare?

Combinatorial Auctions [No Budgets]



- Generate a lot of revenue (e.g., spectrum auctions)
- Important applications (e.g., airport slot auctions)

Social Welfare (SW) as the Efficiency Metric:

$$SW = \sum_{i=1}^{n} v_i(S_i)$$

Truthful mechanisms for maximizing SW in CAs truthfully: **extremely challenging!**

- Instead, we resort to **approximations** for specific classes of valuation functions.
- **Submodular** Functions: $S \subseteq T, x \notin T: v(S \cup \{x\}) - v(S) \geq v(T \cup \{x\}) - v(T)$

State-of-the-art approximations of SW

Worst – Case
(no further assumptions apart from submodularity)

- $O(\log m)$ [Krysta & Vocking, 2012]
- $O(\sqrt{\log m})$ [Dobzinski, 2016]

Bayesian
(valuations drawn from known distributions)

- $O(1)$ [Feldman, Gravin, Lucier, 2014; Duetting, Feldman, Kesselheim, Lucier, 2017]

- Posted price mechanisms used in both cases
- Bidders choose bundles through Demand Queries (DQ):

$$S_i := DQ(v_i, U_i, \vec{p}) = \underset{T \subseteq U_i}{\operatorname{argmax}} \{v_i(T) - p(T)\}$$

Combinatorial Auctions [With Budgets]

Notion of welfare efficiency must balance between the bidder's willingness (= *valuation*) and ability (= *budget*) to pay

Liquid Welfare (LW) [Dobzinski & Paes Leme, 2014]

$$LW = \sum_{i=1}^{n} \bar{v}_i = \sum_{i=1}^{n} \min\{v_i(S_i), B_i\}$$

Our Approach

The Core Mechanism [e.g., Krysta & Vocking, '12]

1. Fix bidder ordering π & set $U_1 = U_2 = \cdots = U$
2. Initial prices $\vec{p}^{(1)} = (p_1^{(1)}, \ \ldots, \ p_m^{(1)})$
3. For each bidder $i \sim \pi$ do:
4. Let i choose $S_i := DQ(v_i, U_i, \vec{p})$
5. With prob. q give i: S_i & set: $U_{i+1} = U_i \setminus S_i$
6. Update prices $\vec{p}^{(i+1)} = (p_1^{(i+1)}, \ \ldots, \ p_m^{(i+1)})$

✓Truthfulness (clear, due to choice of S_i from DQ)

✓ Appropriately choosing *initial prices* & *price update rule*:
→ $O(\log m)$ - apx for SW in worst-case
→ $O(1)$ - apx for SW in Bayesian settings

Lemma 1: If v: submodular/XOS → $\bar{v}$: submodular/XOS

Why not change DQ in Step 4: $DQ(\bar{v}_i, U_i, \vec{p})$? **Not** truthful

Definition: Budget Compatible Demand Query (BCDQ):

$$S_i := BCDQ(v_i, U_i, \vec{p}, B_i) = \operatorname{argmax}_{T \subseteq U_i} \{v_i(T) - p(T) | p(T) \leq B_i\}$$

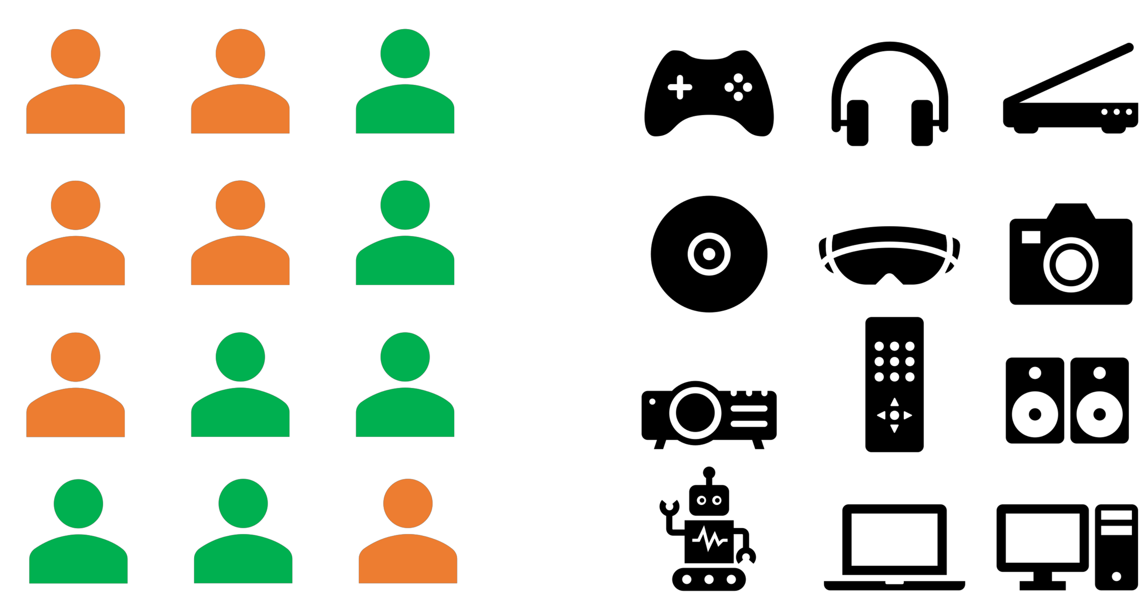
✓Truthfulness **restored**! What about approximations?

Lemma 2: Let $S \subseteq U$: chosen from BCDQ. Then, for any $T \subseteq U$:

1. $\bar{v}(S) \geq \bar{v}(T) - p(T)$
2. $2\bar{v}(S) - p(S) \geq \bar{v}(T) - p(T)$

✓Guarantees same order apx, just added constant of 1/2

Introducing Competitive Markets in CAs



$$\Pr\left[\overline{OPT}_T \geq \left(1 - \frac{\epsilon}{2}\right) \overline{OPT}\right] \geq 1 - \delta$$

✓Homogeneity of the market
✓Important buyers in both sets
✓ With high probability if you only sell in the second set, you don't lose much