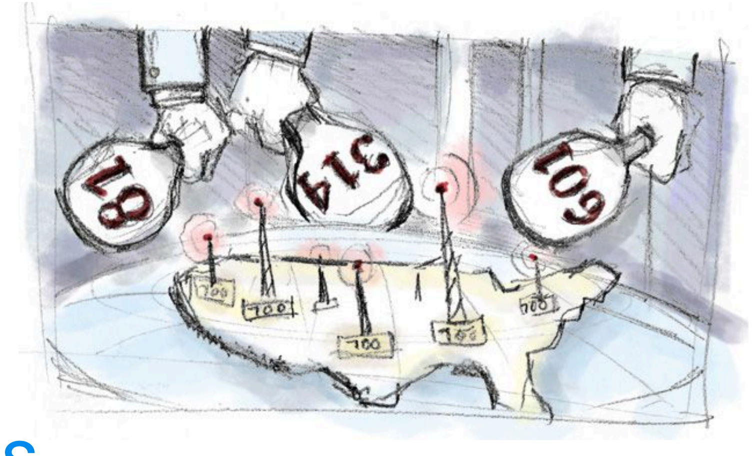
A Bridge between Liquid and Social Welfare in Combinatorial Auctions with Submodular Bidders

Chara Podimata, Harvard University

Dimitris Fotakis (NTUA & Yahoo! Research), **Kyriakos Lotidis** (NTUA) and **Chara Podimata** (Harvard University)

Examples of Combinatorial Auctions (CAs)

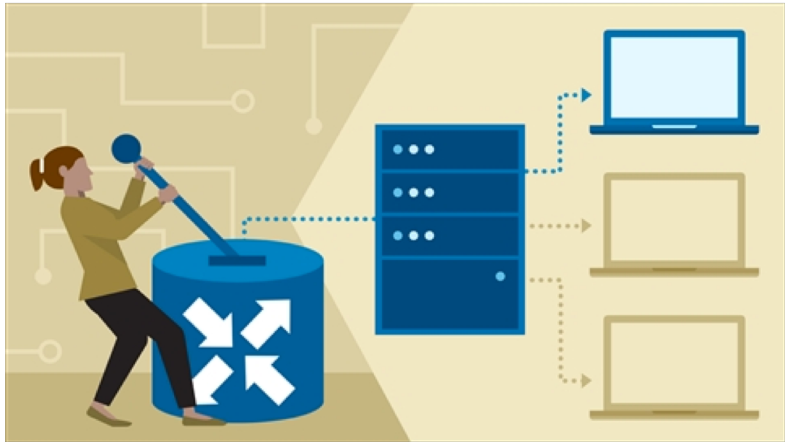
Spectrum Auctions



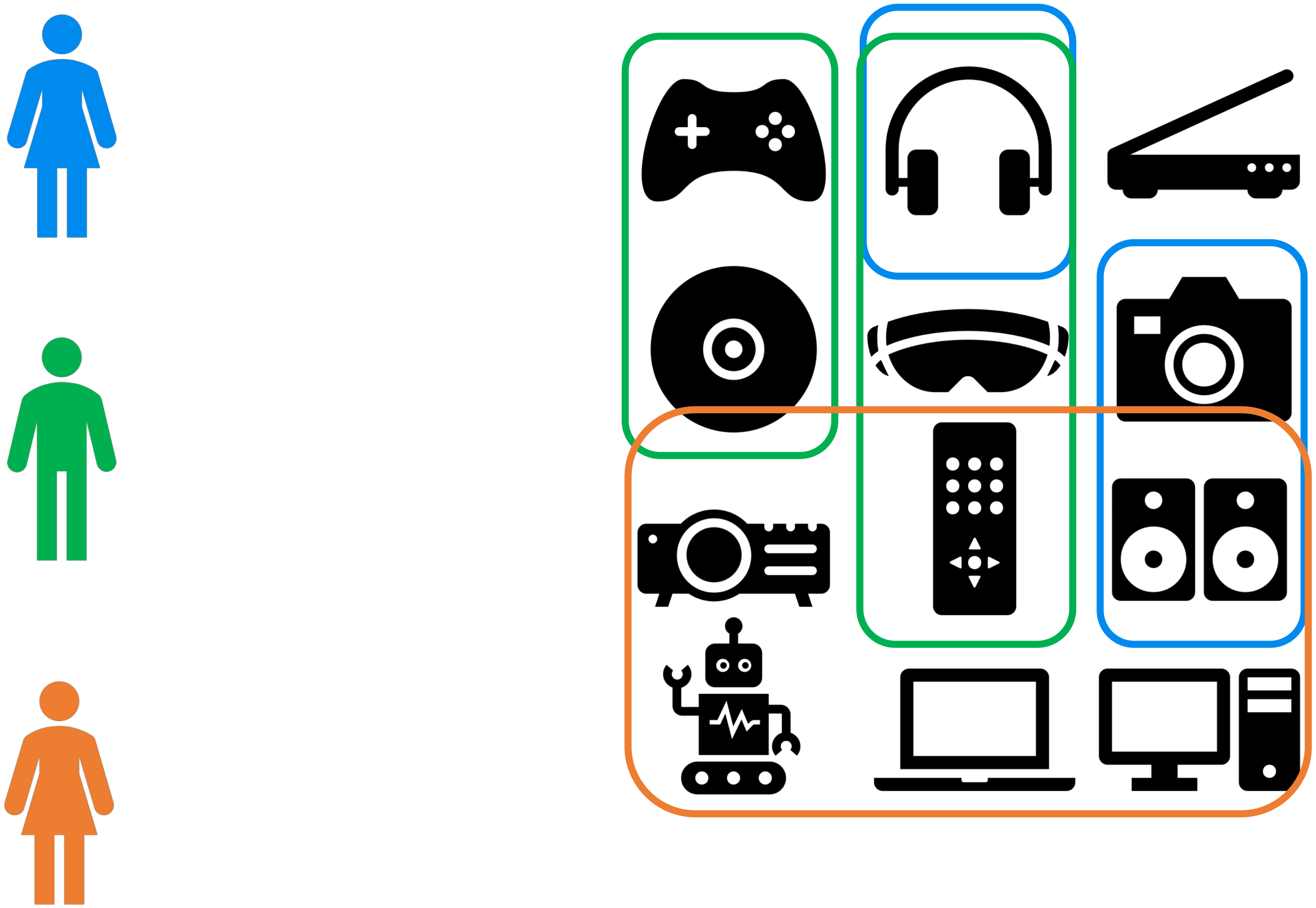
- Canada: ~ $5 billion
- Germany: ~ $57 billion
- India: ~ $13 billion

Airport Slot Auctions

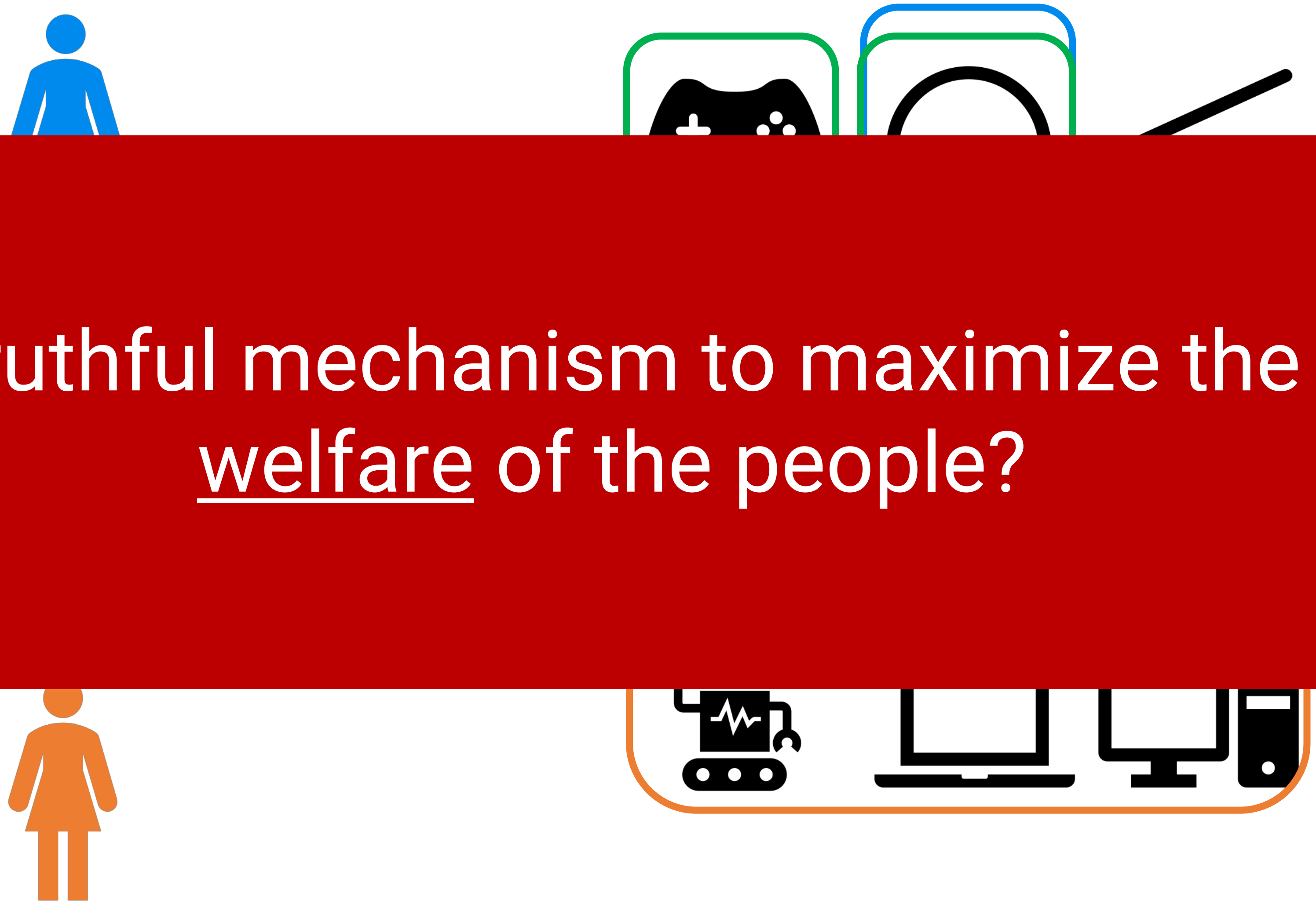
Network Routing Auctions



CAs [No Budgets]



CAs [No Budgets]



Truthful mechanism to maximize the welfare of the people?

Measure of Welfare Efficiency [No Budgets]

Social Welfare: $SW = \sum_{i=1}^{n} v_i(S_i)$

Hard to maximize truthfully in CAs!

⬇

Approximations for specific classes of valuation functions (e.g.,

submodular: $S \subseteq T, x \notin T: v(S \cup \{x\}) - v(S) \geq v(T \cup \{x\}) - v(T)$)

State-of-the-art Approximations for SW in CAs

Worst – Case
(no further assumptions apart from submodularity)

- $O(\log m)$ [Krysta & Vocking, 2012]
- $O(\sqrt{\log m})$ [Dobzinski, 2016]

Bayesian
(valuations drawn from known distributions)

- $O(1)$ [Feldman, Gravin, Lucier, 2014; Duetting, Feldman, Kesselheim, Lucier, 2017]

- Posted price mechanisms used in both cases
- Bidders choose bundles through Demand Queries (DQ):

$$S_i := DQ(v_i, U_i, \vec{p}) = \underset{T \subseteq U_i}{\operatorname{argmax}} \{v_i(T) - p(T)\}$$

Measure of Welfare Efficiency [With Budgets]

[Dobzinski, Paes Leme '14] Liquid Welfare

$$LW = \sum_{i=1}^{n} \bar{v}_i(S_i) = \sum_{i=1}^{n} \min\{v_i(S_i), B_i\}$$

Main Question Addressed

Can we achieve <u>same order approximation</u> for <u>LW</u> with <u>truthful</u> mechanisms, as we did for the SW in CAs w/o budgets?

Main Results

Can we achieve same order approximation for LW

with truthful mechanisms, as we did for the SW in CAs

w/o budgets?

- ✓ Worst – Case: $O(\sqrt{\log m})$ - apx for opt LW
- ✓ Competitive Markets: $O(1)$ - apx for opt LW [Introduced Setting]
- ✓ Bayesian Setting: $O(1)$ - apx for opt LW

Our Approach (1)

The Core Mechanism [e.g., Krysta & Vocking, '12]

1. Fix bidder ordering π & set $U_1 = U_2 = \cdots = U$
2. Initial prices $\vec{p}^{(1)} = (p_1^{(1)}, \ \ldots, \ p_m^{(1)})$
3. For each bidder $i \sim \pi$ do:
4. Let i choose $S_i := DQ(v_i, U_i, \vec{p})$
5. With prob. q give i: S_i & set: $U_{i+1} = U_i \setminus S_i$
6. Update prices $\vec{p}^{(i+1)} = (p_1^{(i+1)}, \ \ldots, \ p_m^{(i+1)})$

✓ Truthfulness
(clear: choice of S_i from DQ)

✓ Appropriate *initial prices* & *price update rule*:
→ $O(\log m)$ - apx for SW in worst-case
→ $O(1)$ - apx for SW in Bayesian settings

Our Approach (2)

Lemma 1. Valuation func is submodular/XOS → liquid valuation ($\bar{v}(S) = \min\{v(S), B\}$) is submodular/XOS

Our Approach (3)

The Core Mechanism [e.g., Krysta & Vocking, '12]

1. Fix bidder ordering π & set $U_1 = U_2 = \cdots = U$
2. Initial prices $\vec{p}^{(1)} = (p_1^{(1)}, \ \ldots, \ p_m^{(1)})$
3. For each bidder $i \sim \pi$ do:
4. Let i choose ~~$S_i := DQ(v_i, U_i, \vec{p})$~~ $S_i \coloneqq DQ(\bar{v}_i, U_i, \vec{p})$
5. With prob. q give i: S_i & set: $U_{i+1} = U_i \setminus S_i$
6. Update prices $\vec{p}^{(i+1)} = (p_1^{(i+1)}, \ \ldots, \ p_m^{(i+1)})$

~~✓ Truthfulness (clear: choice of S_i from DQ)~~

✓ Appropriate *initial prices* & ~~*price update rule*~~ LW:

→ ~~$O(\log m)$ - apx for SW in worst-case~~

→ ~~$O(1)$ - apx for SW in Bayesian settings~~

Our Approach (4)

Definition (Budget Constrained Demand Query):

$$S_i := BCDQ(v_i, U_i, \vec{p}, B_i)$$

$$= \arg\max_{T \subseteq U_i} \{v_i(T) - p(T) | p(T) \leq B_i\}$$

✓ truthfulness

What about the approximation guarantees?

Our Approach (5)

Lemma 2. Let $S \subseteq U$ chosen by BCDQ. Then, for any $T \subseteq U$:

1. $\bar{v}(S) \geq \bar{v}(T) - p(T)$
2. $2\bar{v}(S) - p(S) \geq \bar{v}(T) - p(T)$

Our Approach (6)

The Core Mechanism [e.g., Krysta & Vocking, '12]

1. Fix bidder ordering π & set $U_1 = U_2 = \cdots = U$
2. Initial prices $\vec{p}^{(1)} = (p_1^{(1)}, \ \ldots \ , \ p_m^{(1)})$
3. For each bidder $i \sim \pi$ do:
4. Let i choose $\boldsymbol{S_i} := \boldsymbol{BCDQ(v_i, U_i, \vec{p}, B_i)}$
5. With prob. q give i: S_i & set: $U_{i+1} = U_i \setminus S_i$
6. Update prices $\vec{p}^{(i+1)} = (p_1^{(i+1)}, \ \ldots \ , \ p_m^{(i+1)})$

✓ Truthfulness

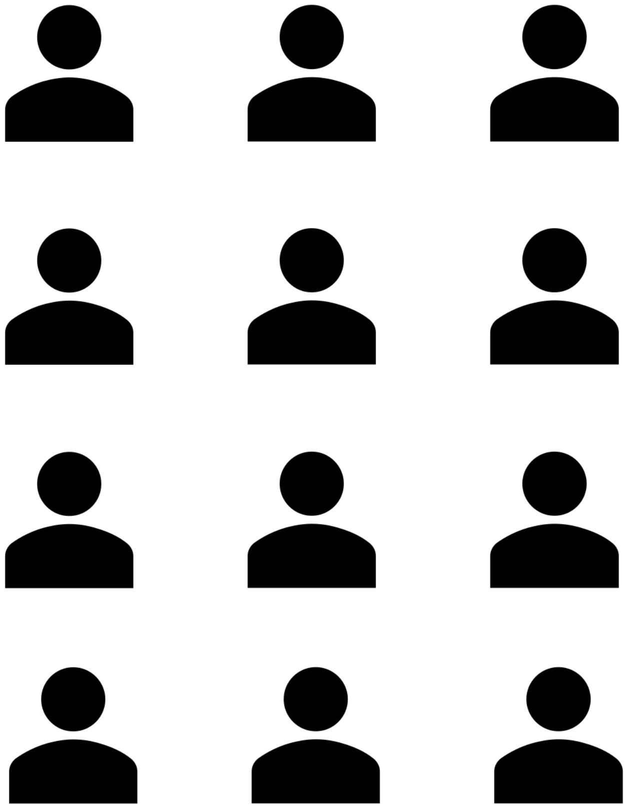
✓ Appropriate *initial prices* & *price update rule*:

→ $O(\log m)$ - apx for **LW** in worst-case

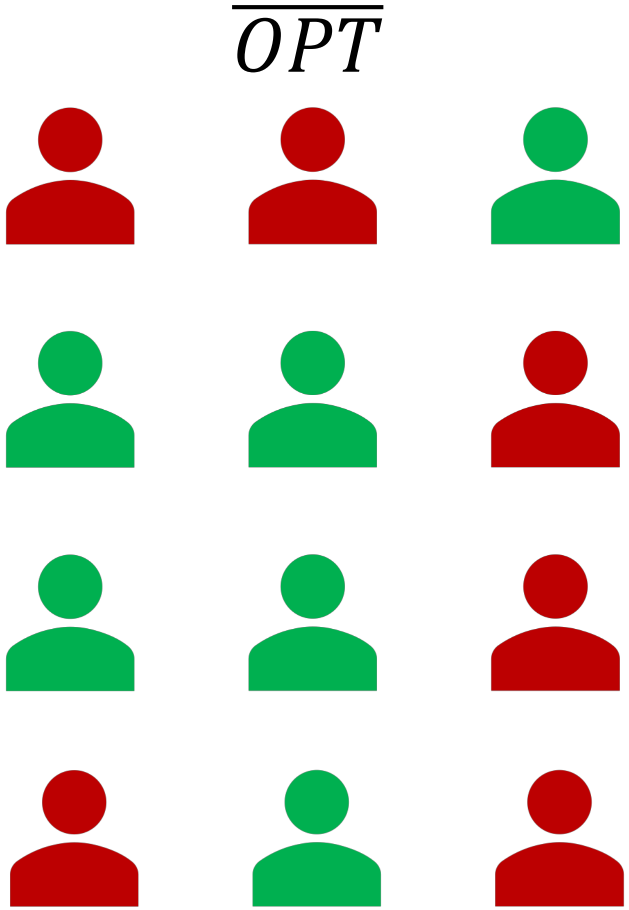
→ $O(1)$ - apx for **LW** in Bayesian settings

Competitive Markets for CAs

$\overline{OPT}$



Competitive Markets for CAs



$$\Pr[\overline{OPT_T}] \geq \left(1 - \frac{\epsilon}{2}\right) \overline{OPT} \geq 1 - \delta$$

Conclusion

Results

✓ Truthful mechanisms for submodular CAs with budgets:

- $O(\log m)$ - apx to opt LW for worst - case
- $O(1)$ - apx to opt LW for Bayesian case
- $O(1)$ - apx to opt LW for Competitive Markets

Future Work

✓ Use of BCDQ & our techniques for extending

results of CAs w/o budgets to budgeted settings


Thank You!